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The Discrete Ordinates (S_N) Method and Lie Symmetries

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X Theoretical Design
Nuclear Threat Assessment

²**Michigan State University**

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Summer Fun



Student Name (Group e.g. NEN-1)

- Educational Background
 - BS Temple University, 2014
 - PhD Michigan State University, Ongoing
- X Theoretical Design
 - Nuclear Threat Assessment
 - Joe Schmidt
- Research
 - Building a 1-D discrete ordinates code and learning about Lie symmetries
 - Studying the role of cross-shell excitations in ^{70}Ni



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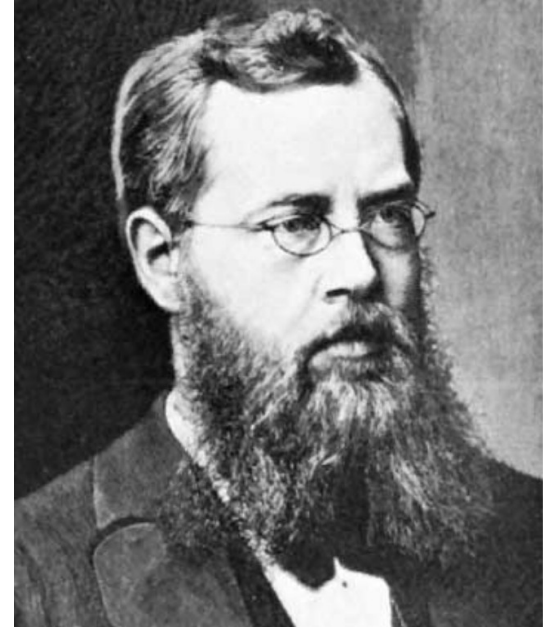
Research Overview and Motivation

Q: Verification and validation are required to understand if a simulation is working, but how do we probe the assumptions underlying the simulation?



Research Overview and Motivation

- A: One way is to check how the assumptions affect the symmetries of the governing differential equations.
- Sophus Lie developed symmetry methods for handling differential equations in the 1870s.



1-D Neutron Transport in Slab Geometry

$$\mu \frac{\partial \psi(x, \mu)}{\partial x} + \sigma(x) \psi(x, \mu) = \sum_{l=0}^L (2l + 1) P_l(\mu) \sigma_l(x) \phi_l(x) + s(x, \mu)$$



- Flux being absorbed in the medium



- Flux scattering into direction μ_n



- External sources and fission



This equation is hard to solve. So let's just solve it for specific angles μ .

Discrete ordinates (S_N) method

We choose a specific set of ordinates μ_n for $n = 1, 2, \dots, N$. This is the

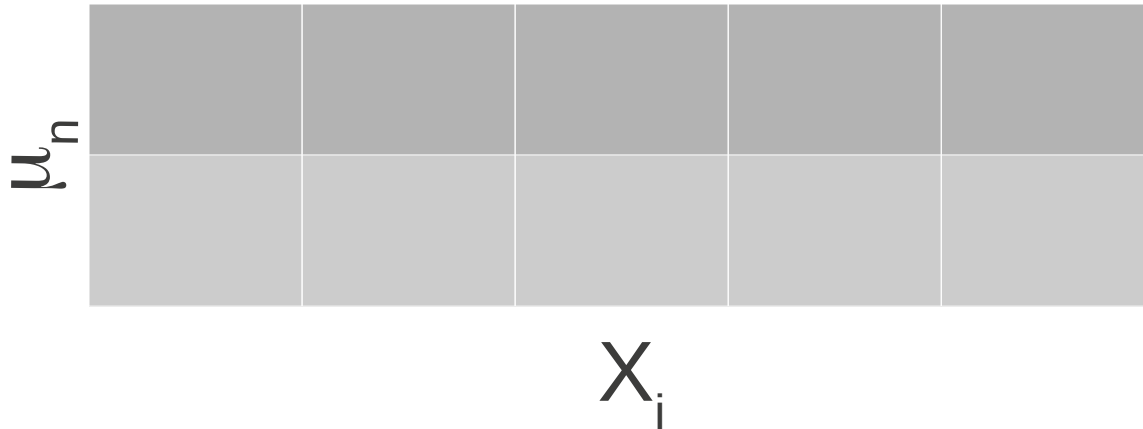
$$\mu_n \frac{d\psi_n(x)}{dx} + \sigma(x)\psi_n(x) = \sum_{l=0}^L (2l+1)P_l(\mu_n)\sigma_l(x)\phi_l(x) + s(x, \mu_n)$$

The set of ordinates is chosen to accurately integrate the scalar flux:

$$\phi(x) = \frac{1}{2} \int_{-1}^1 d\mu \psi(x, \mu) \quad \Rightarrow \quad \phi(x) = \frac{1}{2} \sum_{n=1}^N w_n \psi_n(x)$$

Discrete ordinates (S_N) method

We then discretize our spatial variable.



We can relate the flux at each grid point with neighboring grid points.



- Learned about the discrete ordinates method, and implemented a 1-D slab geometry neutron transport solver in python using this method.
- Started learning about Lie symmetries, though no results yet.